

Reeb Graphs over Point-Set Surfaces

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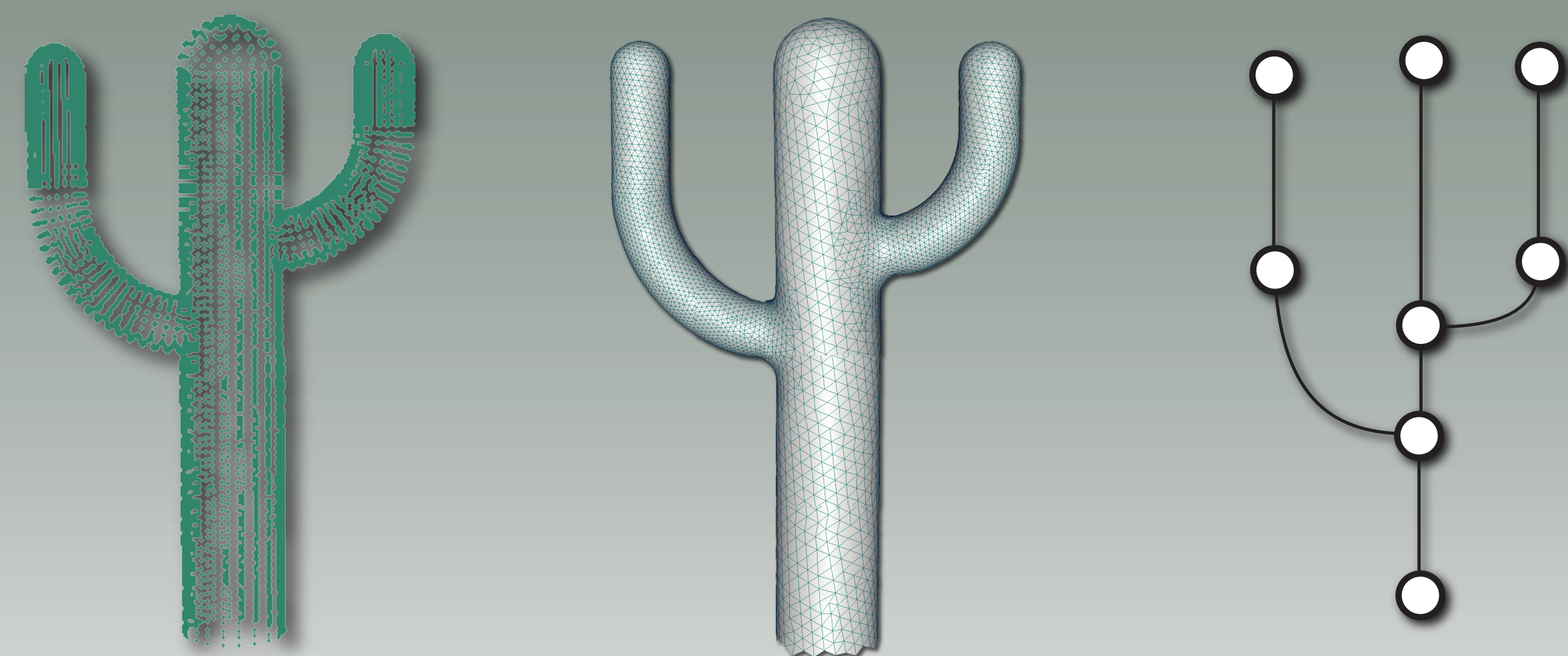
Abstract

We are investigating techniques to compute Reeb graphs over Point Set Surfaces (PSS). PSSs are very popular in computer graphics, but our theoretical understanding is still an active research area. A Reeb graph is a tool in Morse theory that can be used to distinguish topological invariants of surfaces. They also aid in the visualization of geometrical and topological features of surfaces. We use a recently published P.L. reconstruction algorithm for Point Set Surfaces and a novel Reeb graph data structure for the computation. We will investigate the effect of different Morse functions and PSS formulations and the resulting Reeb graphs, hopefully increasing our understanding of Point Set Surfaces.

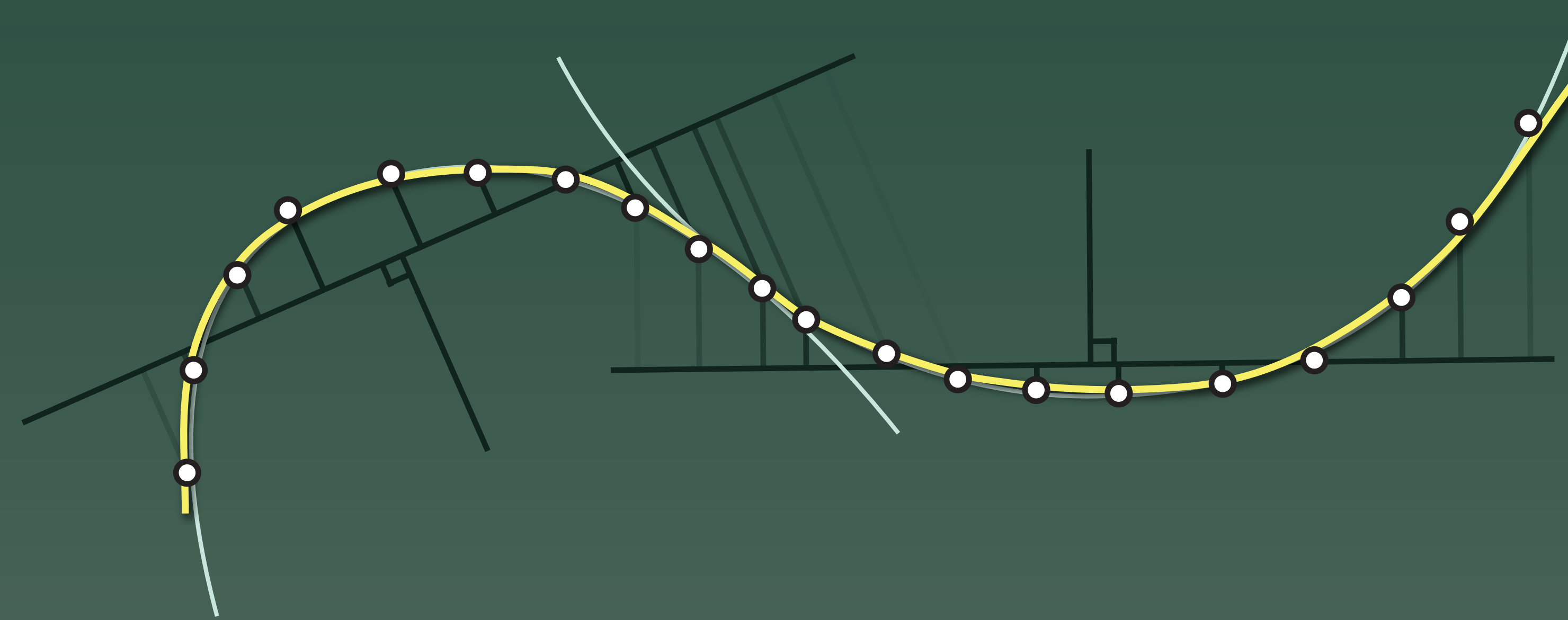
Method

PSS are defined as the fixpoint of a projection operator. This is a simple definition, but it lacks local neighborhood information essential for topological computations. We then first reconstruct a piecewise linear 2-manifold from the PSS and then compute the Reeb graph. We are still investigating how to do this efficiently.

We can construct the entire mesh representation for the PSS and then compute the Reeb graph...

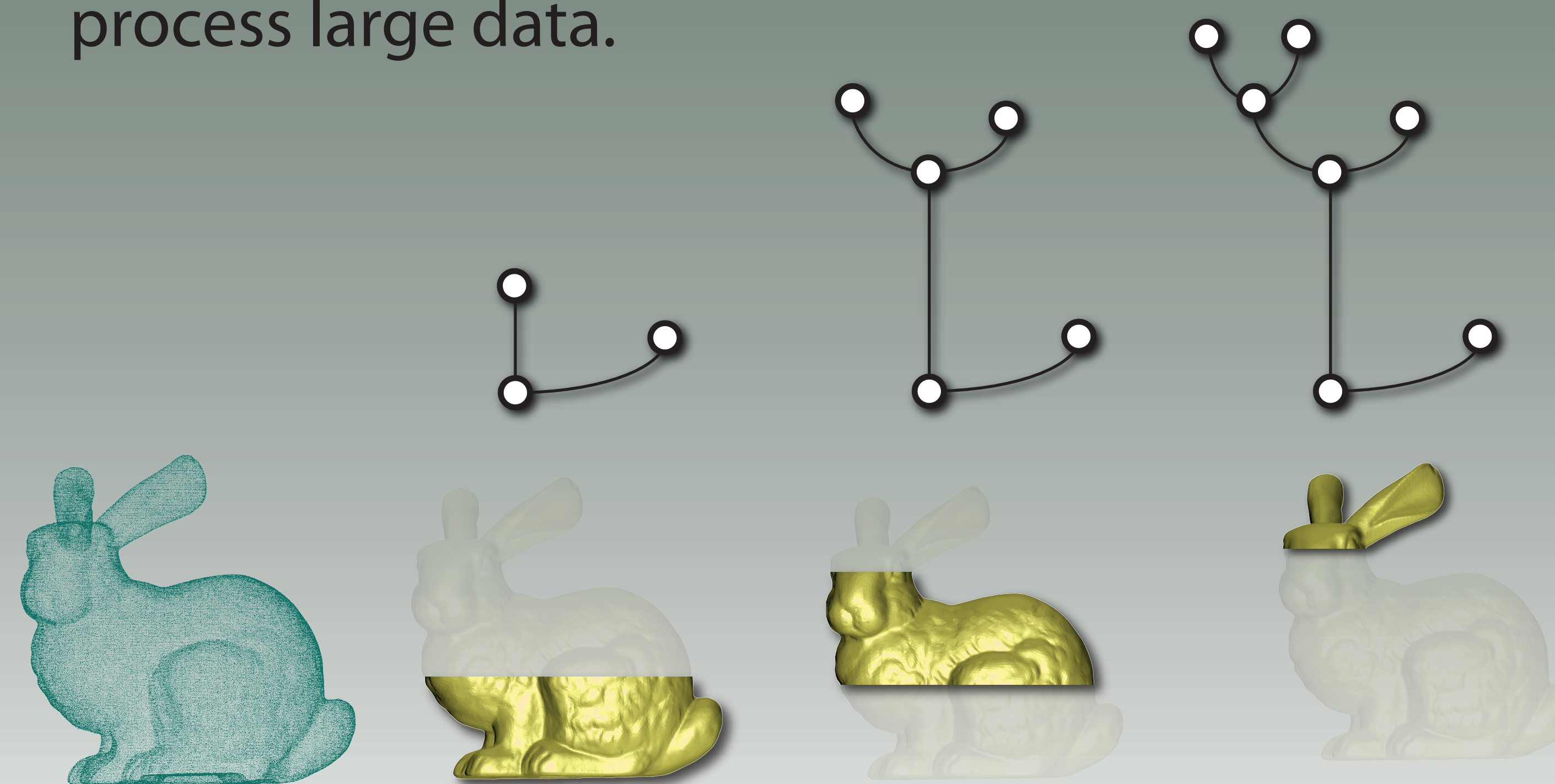


Point Set Surfaces

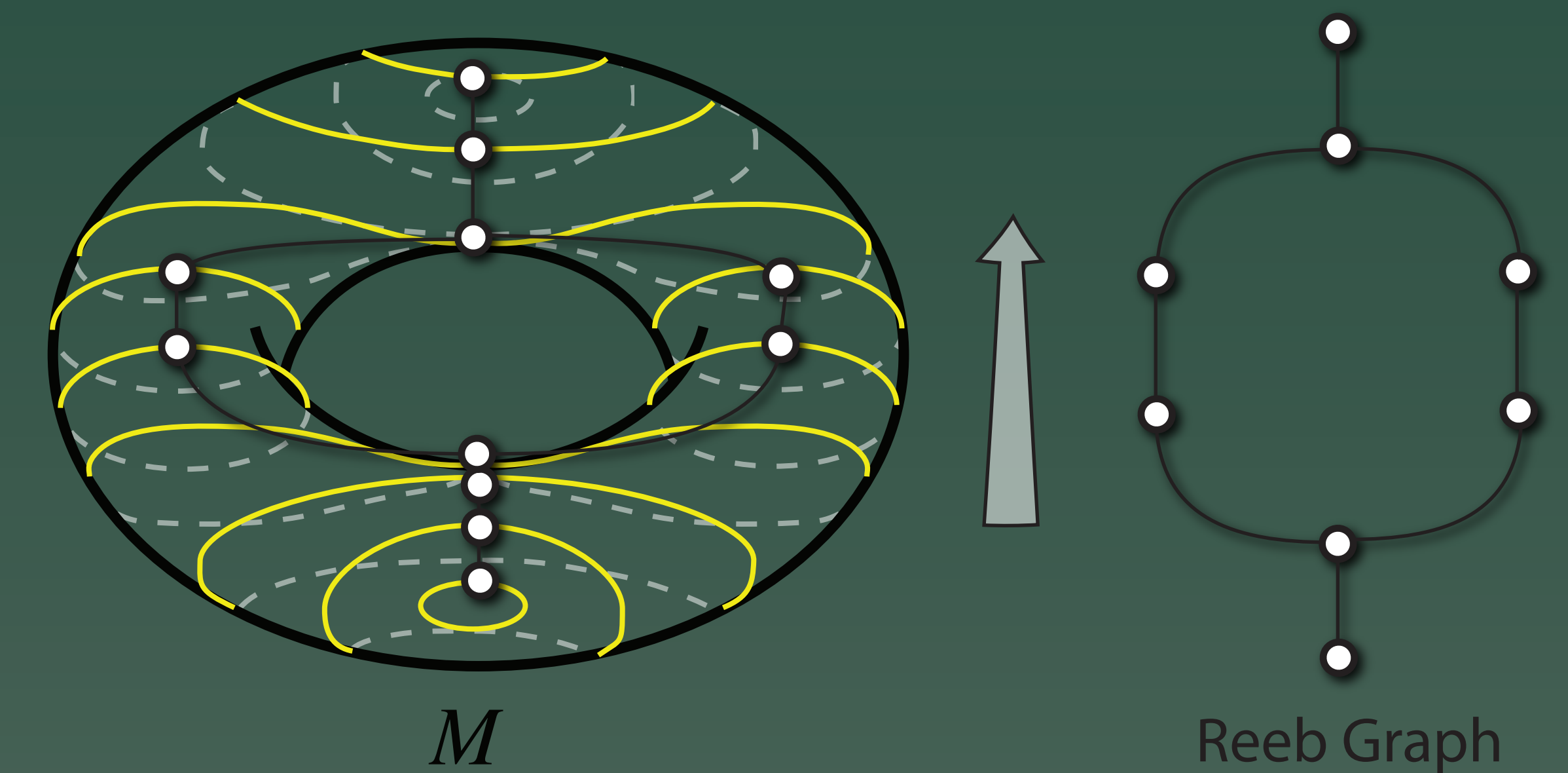


Point Set Surfaces define continuous surfaces from a finite sample of points by approximating functions on local reference frames. Distant points are weighted less through the use of rapidly decreasing weighting functions. The technique is called Moving Least Squares, or MLS. The white curves are the local approximations for the reference frames shown in black, and the yellow curve is the PSS.

...or we can compute the Reeb graph incrementally. This will let us store only a small amount of the mesh in main memory at a time, allowing us to process large data.



Reeb Graphs



Reeb graphs concisely encode the behavior of a Morse function $f: M \rightarrow \mathbb{R}$ by contracting each connected component of a level set to a point. All critical points of the function are nodes on the Reeb graph, giving a rich structure for analysis or visualization.

Discussion

We are still implementing the technique. We expect good performance in practice because all of the operations are very local in nature. We plan to extend the algorithm for out-of-core data by using streaming techniques.